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POST-EINSTEIN TIME DILATION EQUATION IN GRAVITATIONAL FIELD OF STATIC HOMOGENOUS SPHERICAL MASS

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Abstract Einstein's geometrical theory of gravitational is the most accredited theory for explaining time dilation. In this paper the General Dynamical Theory of Gravitation and extended Newtonian gravitational scalar potential exterior to static homogeneous spherical mass is used to study the gravitational time dilation. A new extended equation with more correction terms for gravitational time dilation was obtained. The obtained extended expression for gravitational time dilation reduces to exactly pure Einstein expression to the order of c^0 and to the order of c^{-2} it contains additional correction terms. The significance of the results obtained is that they reduce to the well - known results in weak field approximations. This indicates that the Riemannian extension is mathematically sound and agrees with well-known astrophysical facts. The new expression for the gravitational time dilation can be incorporated in the design of global positioning systems to improve their precision rate.

Keywords: Einstein Theory, General Theory, Newtonian Theory, Scalar Potential, Spherical Mass, Time Dilation and Riemannian Theory.

Introduction

Gravitational time dilation is the actual difference of elapsed time between two events as measured by observers situated at varying distances from a gravitating mass. Newtonian classical Physics does not predict time dilation between observers and both time and space are assumed be absolute to all observers (Einstein, 2004; Howusu, 2013).

In 1905 Einstein presented his theory of Special Relativity in which time was predicted to be relative between observers depending on their relative velocity and the consistency of the speed of light (Einstein, 2004; Bergman, 1987; Weinberg, 1972; Anderson, 1967).

In 1915 Einstein presented his theory of General Relativity and introduced the concept of mass - energy causing a curvature in space time. The first exact solution to Einstein's field equation was the Schwarchild's metric for a non - rotating, non - charged mass (Einstein, 2004; Bergman, 1987; Weinberg, 1972; Anderson, 1967). The first exact solution to Einstein field equation was constructed by Schwarchild's in 1916. Schwarchild's solution accurately predicted the perihelion advance of mercury, light curvature (deflection of light) and time dilation due to energy (mass and momentum) (Einstein, 2004; Bergman, 1987; Weinberg, 1972; Anderson, 1967).

In the year 1991, Howusu postulates the General Dynamical Theory of Gravitation. He postulated that the Instantaneous inertia mass m_i, the passive mass m_p and active mass m_A of a photon of frequency v in all inertial frames and proper time is given by

$$m_A = m_p = m_I = \frac{hv}{c^2} \qquad (1)$$

where, c is the speed of light in vacuum and h is the plank's constant.

This Theory resolves the phenomena of gravitational spectral shift and gravitational deflection of light in the solar system as excellently as Einstein's Geometrical Theory of Gravitation (General Relativity) (Einstein, 2004; Bergman, 1987; Weinberg, 1972; Anderson, 1967).

In our previous paper (Lumbi *et al*., 2014), we applied the Riemannian geometry of space - time to obtain an extended or generalized Newtonian dynamical field equation and scalar potential exterior and interior for static homogeneous spherical fields.

In this article the General Dynamical Theory of Gravitation and the extended Newtonian dynamical gravitational scalar potential exterior to static homogeneous spherical mass was used to substantially extend the gravitational time dilation equation.

1.0 Theoretical Analysis

Consider a photon with instantaneous frequency v in a gravitational field of scalar potential Φ_g. .Then as is well known, its instantaneous kinetic energy T is given by (Atwarer, 1974; Eddington, 1923; Schwarzschild, 2008; Foster & Nightingale, 1979)

$$T = hv \qquad (2)$$

where v is the instantaneous frequency and vanishes when the photon is at rest. Also by definition and postulate in section (1) the instantaneous gravitational potential energy V_g of photon is given by (Atwarer, 1974; Eddington, 1923; Schwarzschild, 2008; Foster & Nightingale, 1979).

$$V_{g=}\frac{hv}{c^2}\Phi_g \qquad (3)$$

It therefore follows by definition that the instantaneous mechanical energy E of the photon is given by (Eddington, 1923; Schwarzschild, 2008).

$$E = hv\left[1 + \frac{1}{c^2}\Phi_g\right] \qquad (4)$$

Therefore, it follows from the principle of Conservation of mechanical energy in gravitational fields that the instantaneous frequency of the photon is given by (Atwarer, 1974; Eddington, 1923).

$$v[r_1 t] = \left\{1 + \frac{1}{c^2}\Phi_g[r,t]\right\}\left[1 + \frac{1}{c^2}\Phi_g[r,t]\right]^{-1} v_0[r_0 t] \qquad (5)$$

where, v is the frequency at the particular position r_0 at time t_0. This is the General Dynamical Law of Gravitational spectral shift for photons.

It follows from the General Dynamical Law of Gravitational spectral shift that the General Dynamical Law of Gravitational time dilation is given by (Foster & Nightingale, 1979).

$$dt(r) = \left\{1 + \frac{1}{c^2}\Phi_g(r)\right\}\left\{1 + \frac{1}{c^2}\Phi_g(r)\right\}^{-1} dt_0(r_0) \qquad (6)$$

The extended Newtonian dynamical gravitational scalar potential exterior based on Riemannian geometry of space - time is given explicitly by (Howusu, 2010).

$$\Phi_g(r) = \frac{GM}{r}\left[1 - \frac{GM}{C^2 R}\right] - \frac{G^2M^2}{C^2 r^2} \qquad (7)$$

where, G is the universal gravitational constant, M is the mass of the planets, R is the radius of the planets and r is the radial

Putting equation (7) into (6) and simplifying yields

$$dt(r) = \left[1 + \frac{1}{c^2}\left(\frac{GM}{r} - \frac{G^2M^2}{RC^2 r} - \frac{G^2M^2}{C^2 r^2}\right)\right]\left[1 + \frac{1}{c^2}\left(\frac{GM}{r} - \frac{G^2M^2}{RC^2 r} - \frac{G^2M^2}{r^2}\right)\right]^{-1} dt_0(r_0) \qquad (8)$$

Supposing we choose a particular point at infinite distance from the body

$$r = \infty \qquad (9)$$

The proper time is given by

$$dt(r) = \left[1 + \frac{1}{c^2}\left(\frac{GM}{r} - \frac{G^2M^2}{RC^2r} - \frac{G^2M^2}{r^2}\right)\right]^{-1} dt_0(r_0)$$
(10)

Equation (10) is an extended time dilation based on General dynamical law of Gravitation and extended Newtonian dynamical gravitational scalar potential exterior based on Riemannian geometry of space - time.

It is interesting and instructive to note that this equation reduces to exactly pure Einstein's expression to the order of c^0 and to the order of c^{-2}. It contains additional correction terms not found in Einstein's expression.

Therefore, the relation between the coordinate and proper time in gravitational field exterior to a static homogeneous spherical body according to the General Dynamical Law of Gravitational time dilation is given by

$$\frac{dT}{dt} = \frac{1}{c^2} \left[-\frac{GM}{r} + \frac{G^2M^2}{r^2}\right]^{-1}$$
(11)

This expression unlike Einstein's expression and General Dynamical Theory of Gravitation contain additional correction terms not found in Einstein's expression and the General Dynamical Law of Gravitational time dilation. These additional correctional terms are open up for theoretical development and experimental investigation and possible applications.

Table 1: Calculated Values of the Ratio of Coordinate Time to Proper Time for Einstein's Theory of Gravitation (General Relativity), General Dynamical Theory of Gravitation and Riemannian Dynamical Theory Gravitation

Body	Mass (M) (kg)	Orbital Radius R (M)	Einstein's Theory of Gravitation $\left[\frac{dT}{dt}-1\right]=\left[\frac{2GM}{c^2R}\right]^{\frac{-1}{2}}-1$	General Dynamical Theory of Gravitation $\left[\frac{dT}{dt}-1\right]=\left[\frac{2GM}{c^2R}\right]^{\frac{-1}{2}}-1$	Riemannian Dynamical Theory of Gravitation $\frac{dT}{dt}=\frac{1}{c^2}\left[-\frac{GM}{r}+\frac{G^2M^2}{r^2}\right]$
Sun	1.960×10^{30}	6.96×10^{8}	2.105×10^{-6}	2.105×10^{-6}	2.088×10^{-6}
Mercury	3.360×10^{20}	5.79×10^{10}	4.300×10^{-15}	4.300×10^{-15}	4.300×10^{-18}
Venus	4.920×10^{24}	1.08×10^{11}	3.376×10^{-14}	3.376×10^{-14}	3.370×10^{-14}
Earth	6.000×10^{24}	1.49×10^{11}	2.984×10^{-14}	2.984×10^{-14}	2.984×10^{-14}
Mars	6.600×10^{23}	2.28×10^{11}	2.145×10^{-15}	2.145×10^{-15}	2.145×10^{-15}
Jupiter	1.920×10^{27}	7.78×10^{11}	1.829×10^{-12}	1.829×10^{-12}	1.710×10^{-12}
Saturn	5.700×10^{26}	1.43×10^{12}	2.954×10^{-13}	2.954×10^{-13}	2.954×10^{-13}

Uranus	9.000×10^{25}	2.87×10^{12}	2.324×10^{-14}	2.324×10^{-14}	2.324×10^{-14}
Neptune	1.020×10^{26}	4.50×10^{12}	1.680×10^{-14}	1.680×10^{-14}	1.679×10^{-14}
Pluto	4.800×10^{24}	4.80×10^{12}	6.050×10^{-16}	6.050×10^{-16}	6.430×10^{-16}

Result on Table 1 shows that the Riemannian Dynamical Theory of Gravitation is in perfect agreement with the Einstein's Geometrical Theory of Gravitation and General Dynamical Theory of Gravitation. It is obvious that to the order of c^{-2} the Riemannian Dynamical Theory of Gravitation contain additional correction terms not found in Einstein's expression and the General Dynamical Law of Gravitational time dilation.

Conclusion

We have shown how to drive an extended time dilation equation using the General Dynamical Theory of Gravitation and the extended Newtonian dynamical gravitational scalar potential exterior to static homogeneous spherical mass. Our results show that the Riemannian Dynamical Theory of Gravitation can be used to satisfactorily explain the time dilation as excellently as Einstein's Geometrical Theory of Gravitation and the General Dynamical Theory of Gravitation. The new expression for the gravitational time dilation can be incorporated in the design of global positioning systems to improve their precession rate.

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